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CRITICAL CURRENT DENSITY IN RAILGUN ACCELERATORS WITH COMPOSITE ELECTRODES

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In analyzing the physical factors limiting the operation of a railgun accelerator with plasma armature it is convenient to compare them using the concept of critical current density I_*/b (I_* is the current in the circuit, b is the electrode width) above which these factors start to manifest themselves. The authors of [1] were probably the first who paid attention to this fact. The estimates of [1, 2] show that the lowest value of I_*/b is obtained with the restriction that the current flowing in the circuit should not result in melting of the electrodes and, as a consequence, in high erosion.

The estimates of I_*/b obtained for monolithic elemental conductors appeared to be so low that they call into question the idea of attainment of high velocities in this acceleration scheme or at least in a simple one-modular accelerator operating by the scheme.

In analyzing the ways of increasing I_*/b, the question arises as to whether the use of composites for this purpose is promising. The known high erosion resistance of composites in pulsed-power opening switches (1.5–3 times as high as that of tungsten) under the assumption that the plasma "arc" armature in a railgun accelerator interacts with the electrodes in much the same manner as in high current opening switches may be regarded as the prerequisites for increasing I_*/b [3]. Experimental application of the electrodes cladded by the high-melting materials W–Cu, W/Re–Cu, Mo–Cu, etc. in railguns was reported in a number of papers [4–8]. It was pointed out that coated copper electrodes (W–Cu, W/Re–Cu) are much better than uncoated copper electrodes for similar railgun arc conditions. It should be noted that, thus far, experimental studies on composite electrodes have not attained a level at which broad generalizations can be made. The results of the majority of experiments are of local character. The behavior of composite electrodes under the action of high-voltage electric discharges has not been analyzed theoretically.

The present paper is intended to study the possibilities of increasing the critical current density in railgun accelerators using composite electrodes of various structure [Fig. 1: homogeneous electrode (a), coated electrode (b), electrode with vertical arrangement of the metal layers *1*, *2* (c), and composite electrode made of a compacted mixture of powders (d)]. Before proceeding to the analysis this way, it should be noted that the requirements for materials selected for the rails go beyond the values of the current density. In real practice account should be taken of the technological problems concerned with the production of the electrodes, as well as of those concerned with the railgun performance, including the multishot life.

1. Statement of the Problem. Let us analyze the ways of increasing the critical current density by the use of composite electrodes using, as an example, the traditional scheme of a railgun accelerator of solid bodies with a plasma armature.

Let us introduce a coordinate system relative to one of the electrodes so that the $\mathbf{x}, \mathbf{y}$ axes are in the plane of its surface and are directed along and across the direction of the plasma armature motion, while the $\mathbf{z}$ axis is directed normally to the plane of its surface.

Let us assume that the temperature of the electrode changes only under the effect of a heat flux from the plasma. As shown in [2], neglect of temperature change due to Joule heating leads to error in determining the surface temperature, which usually does not exceed several percent. The problem of determining the temperature in some vicinity of the point x at the electrode surface may be considered as the problem of

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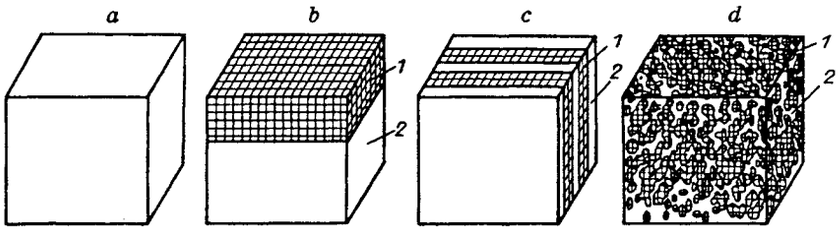

Fig. 1

heating of a half space $z \geqslant 0$ with nonuniform thermal properties by a heat flux q acting for a time Δt equal to the time of passage of the plasma armature over the point x, and is reduced to the solution of the thermal conductivity equation:

$$\rho c \frac{\partial T}{\partial t} = \operatorname{div}(k \operatorname{grad} T), \quad T = T_0, \quad t = t_0(x),$$
$$-k \frac{\partial T}{\partial z}\Big|_{z=0} = q, \quad T\Big|_{z\to\infty} = T_0, T \leqslant T^*, \quad t_0(x) < t \leqslant t_0(x) + \Delta t(x). \tag{1.1}$$

Here it is assumed that in the general case the density ρ, heat capacity c, and thermal conductivity k may be piecewise continuous functions of the point x, y, z depending on temperature; T_0 is the initial temperature; $t_0(x)$ is the time of arrival of the plasma piston at the point x. The constraint $T \leqslant T^*$ is the additional requirement implying that during acceleration the temperature at any point of the electrode surface should not exceed a certain critical temperature of the electrode material (the melting temperature for homogeneous material, the temperature of melting or evaporation for one of the composite components).

Following [2, 9], we make a series of assumptions on the properties of plasma armature and the character of its interaction with the electrode surface. Let us consider that the armature moves as a solid body with constant mass, length l, and electric resistance r. Let us neglect the variation of the internal thermal energy in the plasma piston and assume that all energy dissipating in it radiates uniformly through the surface limiting the volume occupied by plasma. In this case all emitted energy is completely absorbed in the channel of the railgun accelerator as if the emission had happened in a vacuum.

These assumptions make it possible to establish a simple connection between the total current I through the plasma armature and the intensity of heat flux q from its surface

$$q = \frac{rI^2}{S} \tag{1.2}$$

(S is the area of the plasma piston surface).

The dynamics of motion of the plasma piston and the projectile is determined by integrating the equations of motion:

$$m \frac{dv}{dt} = \frac{\lambda}{2} I^2, \tag{1.3}$$

where λ is the inductance per unit length, and m is the sum of the masses of the plasma and the projectile. The dependence of the current density on time or distance L covered by the plasma armature may be determined from joint solution of the Eqs. (1.1)–(1.3). Even in the simplest case (a homogeneous electrode with constant thermophysical parameters) this problem can be solved only numerically.

2. Scaling Ratios. Let us divide the electrode in the direction of motion x into equal intervals with length l, equal to that of the plasma armature. Let us consider the total current I_i to be constant, when the arc moves for the time Δt_i between the points x_i and x_{i+1} which bound the ith interval, the heat flux at the point x_i being also constant. Let us assume that the temperature of the electrode surface does not exceed T^* at the points x_i only.

Taking into account the assumptions made in (1.2), we obtain for the variation of velocity and kinetic

energy at the ith interval

$$v_{i+1} - v_i = \frac{\lambda}{2m} I_i^2 \Delta t_i; \tag{2.1}$$

$$m \frac{v_{i+1}^2 - v_i^2}{2} = \frac{\lambda}{2} I_i^2 l. \tag{2.2}$$

If the heat flux is constant, the temperature change at a certain characteristic point of the surface up to T^* is connected with the amplitude and duration of the heat flux by the relationship [following from the similarity principles for the problem (1.1)]:

$$q_i \sqrt{\Delta t_i} = K(T^* - T_0, \eta_i), \tag{2.3}$$

where $\eta_i = \delta_i q_i / k_s T^*$ is a dimensionless parameter, δ_i is the characteristic size of spatial inhomogeneities in the electrode structure at the point x_i; and k_s is the scale value of the thermal conductivity coefficient.

The right-hand side of the expression (2.3) is independent of the amplitude of the heat flux pulse, if any of the following conditions holds: 1) the medium is spatially homogeneous, 2) the characteristic size of the spatial inhomogeneities changes as the plasma piston accelerates so that $q_i \delta_i = \text{const}$, or 3) $\delta_i \ll k_s T^*/q_i$ ($\eta = 0$). If any of the above conditions hold and the right-hand side of (2.3) is independent of q, from (1.2) and (2.1)–(2.3) we obtain the equation connecting the velocity values at the points x_i and x_{i+1}:

$$(v_{i+1} - v_i)(v_{i+1}^2 - v_i^2) = C \quad \left(C = 2l \left(\frac{\lambda S}{2mr}\right)^2 K^2\right). \tag{2.4}$$

This equation makes it possible to determine successively all values of v_i starting from v_0. Then the values of I_i, Δt_i, and t_i can be calculated from (2.1) and (2.2).

Apparently, the values v_i' and v_i obtained from the solution of (2.4) with different right-hand sides C' and C, whose ratio equals β, will be connected by the relationship $v_i' = \beta^{1/3} v_i$ if $v_0' = \beta^{1/3} v_0$ or $v_0' = v_0 = 0$. If the parameters of a railgun accelerator λ, l, m, r, and S involved in the right-hand side of (2.4) may be considered invariant for the electrodes made of different materials, then the higher velocity can be achieved without erosion at the given acceleration distance L using electrodes with higher K. In this case, in accordance with (1.2) and (2.1)–(2.4) we have

$$\frac{v_i'}{v_i} = \frac{t_i}{t_i'} = \frac{I_i'/b}{I_i/b} = \left(\frac{K_i'}{K_i}\right)^{2/3}. \tag{2.5}$$

These similarity relationships allow one to estimate the efficiency of employment of the electrodes of an arbitrary material X as compared to the conventionally used copper ones.

Let us introduce the coefficient α characterizing the relative heat resistance of the material by the formula

$$\alpha = \left(\frac{K_{\mathrm{X}}(T_{\mathrm{X}}^*)}{K_{\mathrm{Cu}}(T_{\mathrm{Cu}}^*)}\right)^{2/3}. \tag{2.6}$$

In this case, using (2.5) and (2.6), we have

$$\left(\frac{I_*}{b}\right)_{\mathrm{X}} = \alpha \left(\frac{I_*}{b}\right)_{\mathrm{Cu}}, \quad v_X(x) = \alpha v_{\mathrm{Cu}}(x). \tag{2.7}$$

We shall consider the melting temperature T_m of a pure metal as its critical temperature. The quantity K can be calculated using the known expression corresponding to the case where a half space whose thermal properties are independent of the temperature is heated by a constant heat flux:

$$K = \sqrt{\pi \rho c k}\,(T_m - T_0)/2. \tag{2.8}$$

In this case, if the values ρ, c, and k of the metal are taken for the temperature $(T_0 + T_m)/2$, then in most cases the error will not exceed 10% [10].

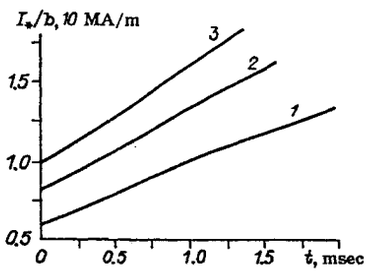

Fig. 2

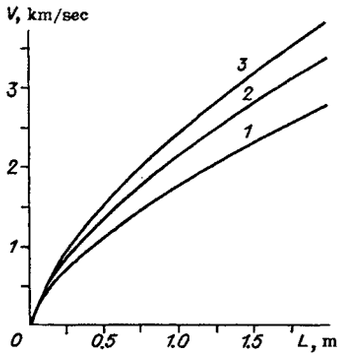

Fig. 3

The dependences $v(L)$ and $I_*(t)/b$ obtained for copper electrodes from (2.4) and (2.8) for $\lambda = 0.3$ mH/m, $r = 1$ mΩ, $m = 1$ g, and channel cross section 1×1 cm are presented in Figs. 2 and 3. Curves 1, 2, and 3 correspond to $l = 5$, 10, and 15 cm. Analogous dependences for an arbitrary material can be plotted, given the coefficient α, from the data presented in Figs. 2 and 3 and the relationship (2.7).

The values of α for a number of metals are presented in Table 1. It is apparent that only electrodes made of W, Mo, Ta, and Re can compete with copper electrodes. However, the use of these materials can be limited by technological considerations (production and treatment difficulties), as well as by a number of additional physical requirements of the electrodes (their ability to withstand high pulsed mechanical and thermal stresses, high electric conductivity, chemical resistance, etc.). Therefore, composite electrodes are of interest due to their higher technological potential and probably higher heat resistance.

3. Composite Electrodes. Let us consider how to increase the electrode heat resistance (which, within the framework of the problem discussed, is equivalent to increasing K) for some of the simplest types of composite structures presented in Fig. 1*b–d*. We assume the thermal contact of the materials to be ideal. The possible chemical reaction between the components of the composite resulting in formation of intermetallic compounds at the interfaces, mechanical movements of different parts of the composite electrode due to its probable partial melting, and the effect of high pulsed mechanical load will be neglected.

A. *Coated electrodes.* The maximum value of K for coated electrodes (Fig. 1*b*) will be achieved at a certain value δ_{opt} which is determined from the condition that the surface and the interface attain simultaneously the temperatures critical for the coating and basic materials. It should be noted that the critical temperature at the material interface can be higher than the melting temperature of the electrode, but a melt layer which may be thicker than the coating will appear under the latter. In this case the destruction of the coating and abrupt ejection of the melt in the interelectrode space are probable. The value δ_{opt} depends also on the amplitude of the heat flux. But, as was mentioned above, under the assumption that, when the plasma armature passes from point x_i to point x_{i+1} , the thickness of the layer changes as $\delta_{i+1} = \delta_i(I_i/I_{i+1})^2$, the value of K will remain invariant and at $\delta_0 = \delta_{\text{opt}}(I_0)$ will will be equal to the maximum value for a coated electrode.

The values of δ_{opt} can be estimated from the formulas:
in the absence of melting

$$\delta_{\text{opt}} = k_1(T_{m_1} - T_{m_2})/q; \tag{3.1}$$

in the presence of melting and provided that the depth of melting of the basic material equals the coating thickness

$$\delta_{\text{opt}} = \frac{k_2 k_1(T_{m_1} - T_{m_2})}{(k_1 + k_2)q}. \tag{3.2}$$

The values of δ_{opt} calculated from (3.1) and (3.2) in which k_1 and k_2 correspond to $(T_{m_1} - T_0)/2$ and T_{m_2} differ from those obtained in numerical calculations of the heat problem (1.1) by no more than 10%.

TABLE 1

Metal	α
W	1.38
Mo	1.17
Re	0.99
Ta	0.99
Nb	0.89
Cr	0.87
Ni	0.78
Fe	0.69
Hf	0.60
Al	0.55

TABLE 2

Bimetal	α_1	α_2
W–Cu	1.443	1.628
Ta–Cu	1.188	1.358
Mo–Cu	1.299	1.445
Re–Cu	1.197	1.344
Cr–Cu	1.124	1.202
Ir–Cu	1.282	1.441
Os–Cu	1.312	1.487
Nb–Cu	1.135	1.280

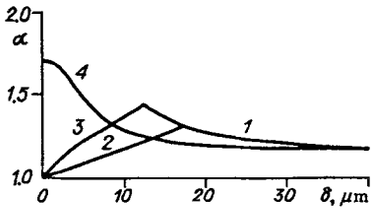

Fig. 4

Figure 4 presents the dependence of the coefficient of relative heat resistance α for copper electrodes with molybdenum coating, Mo–Cu, on the dimensionless thickness of the coating δ when $q = 10^6$ W/cm^2 (lines *1, 2* are for the case without melting of the copper electrode; *1, 3* are for melting to a depth equal to the coating thickness).

Presented in Table 2 are the maximum values of α for copper electrodes coated with different materials calculated on the condition that the temperature at the interface does not exceed the electrode melting temperature (α_1), and for the case where the melt depth under the coating is equal to the coating thickness (α_2).

B. *Vertical arrangement of the layers.* The other simplest type of composite structure is the electrode composed of alternating layers of two materials (1 and 2) with total thickness δ and relative thicknesses of the layers ε and $1 - \varepsilon$ which are placed across the electrode surface (Fig. 1*c*). For definiteness, let the heating rate and melting temperature of the first material be higher than those of the second one. It is evident that, as for the coated electrode, there will appear an additional heat flux from the first material to the second. The maximum temperature at the surface of the first material will be lower compared to the heating of the homogeneous material and achieve its melting temperature at higher K. When ε is fixed this increase of K is proportional to the structural size of δ.

As opposed to the coated electrode, the heating rate of the second material will increase, since the heat flux affecting its surface directly will be joined by the heat flux from the first material. Therefore, the melting temperature of the second material at the point of its maximum temperature, which is at the material interface, will be achieved at lower K. Consequently, for such a structure an increase in K is impossible if both materials are required to remain solid during the heating. However, one can assume that, when the second material melts, the frame of the first material which remains solid will prevent immediate removal of the melt from the electrode surface. Therefore, the melting temperature of the material forming the frame or the evaporation temperature of lower-melting material can be considered as the critical temperature for the structure. In this case, as follows from qualitative analysis, a maximum increase in K will occur as $\delta \to 0$. Figure 4 illustrates this statement. Curve *4* represents the dependence $\alpha(\delta)$ for a layered Mo–Cu structure for $\varepsilon = 0.4$ and $q = 10^6$ W/cm^2, which was obtained in numerical solution of the heat problem (1.1). Depending

TABLE 3

Composite	ε		
	0.25	0.5	0.75
	α		
Re–Cu	1.792 1.417	1.643 1.160	1.253 0.992
Mo–Cu	1.825 1.675	1.622 1.456	1.399 1.288
W–Cu	1.829 1.720	1.632 1.509	1.420 1.339
Ta–Cu	1.781 1.504	1.528 1.237	1.241 1.047
W–Mo	1.428 1.426	1.416 1.413	1.402 1.400
W–Re	1.830 1.126	1.264 1.182	1.330 1.263

on δ, the value K was determined either by the inception of melting of the molybdenum layer (higher values of δ), or by the attainment of the evaporation temperature by the copper layer. It is evident that the maximum $\alpha(\delta)$ is achieved as $\delta \to 0$.

C. *Compacted mixture of powders.* The qualitative conclusions made for the heating of electrodes with vertical arrangement of the layers will hold true for composite materials compacted of powder particles of arbitrary form (Fig. 1*d*).

Therefore, if the material with higher heating rate does not form a continuous coating, the material with lower heating rate will always be overheated, and an increase in K in it may occur only if melting is allowed. Maximum increase in K will be obtained when the maximum size of the particles forming the composite is much less than the size of a thermal skin-layer. The value of K will not depend on the heat flux and may be determined from the solution of a one-dimensional thermal conductivity equation with certain averaged statistical values of thermophysical parameters depending on composition and spatial structure of the compound. In this case one of the composite materials should remain solid during heating, and the rigid frame of its structure should be preserved when the second material melts. In this case the lowest of the temperatures (the melting temperature of the material forming the frame or the evaporation temperature of the low-melting material) will be the critical temperature.

The mean static thermal conductivity coefficient k_c for compacted mixture of two powders with volume content of the components ε and $1 - \varepsilon$ and different thermal conductivity coefficients k_1 and k_2 can be determined only if the specific spatial structure of the compound is known. However, this thermal conductivity cannot be less than the thermal conductivity $k_\perp$ of the layered medium made of the same materials with identical volume content of each material and cannot be greater than $k_{||}$ when the heat conducts across the layers. The expressions for $k_\perp$ and $k_{||}$ are as follows [11]:

$$k_{||} = \varepsilon k_1 + (1 - \varepsilon)k_2; \tag{3.3}$$

$$k_\perp = \left(\frac{\varepsilon}{k_1} + \frac{1 - \varepsilon}{k_2}\right)^{-1}. \tag{3.4}$$

For the volume heat capacity of an arbitrary composite $(\rho c)_c$ the following expression is valid:

$$(\rho c)_c = \rho_1 c_1 \varepsilon + \rho_2 c_2 (1 - \varepsilon). \tag{3.5}$$

The upper and lower limits of the coefficient of relative heat resistance α calculated for $\varepsilon = 0.25$, 0.5, and 0.75 using expressions (3.3) and (3.4) for the coefficient of thermal conductivity are presented in Table 3.

The values were obtained in solving the heat problem (1.1) with the coefficients determined according to (3.3)–(3.5) and taking into account the melting enthalpy at phase transition.

The analysis allows us to conclude that composites hold promise for use as electrodes in railgun accelerators. The problem of optimization of the electrode structure should be studied separately. However, preliminary estimates showed that electrodes produced by compaction of powders may be more promising.

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